■ Mixed Strategy: In mixed strategy, both players A and B select their m and n moves symolteneously. Let us defined a set of ~~n-q~~ m-quantites, $p = \{p_1, p_2, p_3, \dots, p_m\}$. such that $p_i \geq 0$, $i = 1, 2, \dots, m$ and $\sum_{i=1}^{m} p_i = 1$.

Now to obtained a least possible games, the player-A may utilize the services of the moves $A_1, A_2, \dots, A_m$ in such a way that A_i performs p_i times. of the total work so that the total work will be done by the moves $A_1, A_2, \dots, A_m$.

Similarly if $q = (q_1, q_2, \dots, q_n)$. be a set of quantites such that $q_j \geq 0$, $J = 1, 2, \dots, n$ and $\sum_{j=1}^{n} q_j = 1$.

Then B may utilize the ~~sur~~ services of the moves $(B_1, B_2, \dots, B_n)$; $q_1, q_2, \dots, q_n$ times.

of the total work respectively to serve his perpuse ignoring all about the move or moves taken by his opponents A. The quantites p_i and q_j associated with the i^{th} or j^{th} move of A and B respectively are called the probabilities of the respective moves.

~~The qu~~
We now defined two variable vectors $P=(p_1,p_2,\dots,p_m)$ and $q=(q_1,q_2,\dots,q_n)$. it is always possible to determine some perticuler value of p and q ~~such that~~ say p^* and q^* such that the value of the game can be determine.

- **Pay-off function:** Let, $(a_{ij})_{mn}$ be the pay-off matrix for a "two person zero sum game" where maximization player is A and the minimization player be B, then the pay-off function of this game is denoted by $E(p,q)$, which is actually the the mathematical expectati of the game and ~~its~~ is defined as

$$E(P,q) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}p_i q_j \quad , \text{where } p=(p_1,p_2,\dots,p_m) \text{ and } q=(q_1,q_2,\dots,q_n)$$

If, B takes his pure j^{th} move only, then the expected ~~game~~ gain of the player-A is given by

$$E_j(p) = \sum_{i=1}^{m} a_{ij} p_i \quad , \text{for } j=1,2,\dots,n$$

On the other hand, for perticuler i^{th} move of the player-A only, the expected loss of the player B is given by $E_i(q) = \sum_{j=1}^{n} a_{ij} q_j \;, i=1,2,\dots m$

■ Find the value of the 2×2 game algebraically by using mixed strategies.

Ans:

		Player-B B_1	B_2	Row minima
Player-A	A_1	2	3	2
	A_2	4	-1	-1
	Col. maxima	4	3	3 | 2

From the above discussion, it is clear that there is no saddle point for the pure strategies of the players A and B.

∴ To solve the game, we shall use the mixed strategies.

Let, $P=(P_1, P_2)$, where $P_1, P_2 \geq 0$ and $P_1+P_2=1$.

where P_1 and P_2 denotes the probabilities of the mixed strategies for the player-A to use the moves A_1 and A_2 respectively.

Also, let, $q=(q_1, q_2)$, where $q_1, q_2 \geq 0$ and $q_1+q_2=1$.

where q_1 and q_2 denote the probabilities for using the moves B_1 and B_2 of the minimization player-B.

When the player-B use the pure move B_1, the expected gain of the player-A is $E_1(P) = 2P_1+4P_2$

also, if the player-B use the pure move B_2 then the expected gain of the player-A is $E_2(P) = 3P_1 - P_2$.

If the value of the game exist then

$E_1(P) = E_2(P) = V$.

Now, $E_1(P) = E_2(P)$ gives.

⇒ $2P_1+4P_2 = 3P_1 - P_2$

or, $P_1 = 5P_2$

or, $\frac{P_1}{5} = \frac{P_2}{1} = \frac{1}{6}$ $[\because P_1+P_2=1]$.

∴ $P_1 = 5/6$ and $P_2 = 1/6$.

and ~~B~~ $V = 2p_1 + 4p_2 = 2\cdot\frac{5}{6} + 4\cdot\frac{1}{6} = \frac{7}{3}$.

Also, Considering the pure move A_1 of the player-A, the expected loss of the player-B is

$E_1(q) = 2q_1 + 3q_2$. and fore the pure move A_2

of the player-A, $E_2(q) = 4q_1 - q_2$.

If the value of the game exist, then $E_1(q) = E_2(q) = V$

Now, $E_1(q) = E_2(q)$ gives.

$$2q_1 + 3q_2 = 4q_1 - q_2.$$

or, $$2q_1 = 4q_2.$$

or, $$\frac{q_1}{2} = \frac{q_2}{1} = \frac{1}{3}.$$

$\therefore q_1 = \frac{2}{3},\ q_2 = \frac{1}{3}$.

$\therefore V = 2q_1 + 3q_2 = 2\cdot\frac{2}{3} + 3\cdot\frac{1}{3} = \frac{7}{3}$.

Hence, the sol^n of the game is $\overset{*}{p} = (5/6, 1/6)$ and $\overset{*}{q} = (2/3, 1/3)$. and the value of the game is $7/3$ units. (Ans)